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Incorporating Radiative Cooling into a Cosmological Hydrodynamic Code

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ABSTRACT
A possible inconsistency arising when a radiative cooling term is incorporated in a finite resolution self-gravitating hydrodynamic code is discussed. The inconsistency appears when the heating-cooling balance within the cooling and collapsing gas cloud is broken near the resolution limit of a numerical code. As the result, the cooling time of a fluid element increases enormously, leading to the unphysical conclusion that the fluid element does not cool and is therefore stable against the collapse. A special *cooling consistency condition* is introduced which approximately restores the heating-cooling balance and leads to a numerical solution that closely mimics the exact (infinite resolution) solution.

Key words: cosmology: theory – hydrodynamics – methods: numerical

1 INTRODUCTION

Cosmological hydrodynamic simulations are now playing larger and larger role in developing the theory of galaxy formation. They are successfully incorporating more and more complex physical processes that are important in the process of galaxy formation, such as dark matter evolution, gas dynamics, radiative processes, star formation etc. Simulations, therefore, are challenged to solve a very complex system of nonlinear equations incorporating physical processes with widely different characteristic time-scales. However, one usually does not solve the whole system of equations by a single numerical technique (simply because there are no such methods invented yet), but by combining various numerical techniques for separately solving for dark matter, gas dynamics, radiative cooling etc. The advantage of this approach is that one can use known numerical methods that have been developed to separately simulate each of physical processes incorporated into a larger simulation. However, the pay off for using different methods for solving parts of the whole problem is that those methods are not necessarily consistent with each other; in other words, straightforward combining of two of numerical techniques that solve two different sub-sets of the whole system of nonlinear equations may not lead to a techniques that solves the whole system. An example of how direct merging of a gas dynamical solver with a gravity solver can lead to a code that produces unphysical results (energy is not balanced locally) is given in Gnedin & Bertschinger (1996). Therefore, a special *gravitational consistency condition* between the gravity and gas dynamics ought to be satisfied in order to produce physically sensible solutions.

However, the requirement of consistency between numerical methods solving separate parts of the whole system of equations is a general one, and applies not only to the case of gravity and gas dynamical solvers. In particular, in this paper I demonstrate how a *cooling consistency condition* can emerge when radiative cooling is incorporated into a self-gravitating hydrodynamic code.

Radiative cooling is the crucial mechanism responsible for condensation and collapse of baryonic gas into galaxies, and an adequate treatment of it in numerical simulations is required in any realistic cosmological gas dynamical simulation except, perhaps, in a simulation of clusters of galaxies. Yet, since radiative cooling can often introduce into the solution the time-scales that are orders of magnitude shorter than the characteristic time-scale for the gas evolution (the sound crossing time), no numerical scheme has been invented to solve the whole system of equations including gas dynamics and cooling together, but rather two different numerical methods, one for gas dynamics, and another for cooling, are combined together to incorporate radiative cooling into the gas dynamical code (Cen 1992; Katz, Hernquist, & Weinberg 1994; Evrard, Summers, & Davis 1994; Anninos et al. 1994; Gnedin 1995). Therefore, one can expect that consistency between those two different methods may not be automatically achieved.

The cooling inconsistency was largely ignored in the previous numerical work except in a simulation by Katz et al. (1994), who have abandoned the cooling time condition in their three-dimensional simulation of galaxy formation for

all gas elements below 30,000 K; this solution to the cooling inconsistency is acceptable as long as an equilibrium standard cooling curve is used as a cooling function; in a simulation which includes nonequilibrium time-dependent ionization, radiative transfer effects, cooling by heavy elements and/or by molecules, this approach would become inappropriate, since a cooling function is then a function of position and time, and no characteristic temperature like 30,000 K, below which the cooling time condition may be abandoned, can be specified a priori.

This papers is composed as follows. In §2 I utilize the spherically symmetric Lagrangian hydrodynamic code to emphasize the importance and role of the cooling consistency condition, and in §3 I apply those results to correct the existing cosmological hydrodynamic code based on the "Softened Lagrangian Hydrodynamics" method (SLH; Gnedin 1995) for the inconsistency between the hydrodynamic and cooling solvers. I conclude in §4 with brief discussion.

2 COOLING CATASTROPHE IN A SPHERICALLY SYMMETRIC CASE

I first consider a collapse of a spherically symmetric self-gravitating gas cloud which is loosing its energy via radiative cooling. At the initial state the cloud is at rest, has a uniform temperature $T = 1$ (in dimensionless units) and the density profile:

$$\rho = \frac{1}{1+r^2}, \tag{1}$$

where all quantities are dimensionless for simplicity, and the gravitational constant $G = 1$. The gas has a constant polytropic index $\gamma = 5/3$, and equation of state is

$$P = \rho T.$$

The internal energy of the gas is lost due to radiative cooling at a rate

$$\left(\frac{dU}{dt}\right)_{\rm COOL} = -\rho^2 \Lambda(T), \tag{2}$$

where the cooling function $\Lambda(T)$ has the following form:

$$\Lambda(T) \equiv 10^{-2} T^5 \exp(-T^{-4}). \tag{3}$$

This cooling function allows for an analytical solution for the cooling evolution at a time interval short compared to the dynamical time and simultaneously mimics the steep cut-off at 10^4 K in the real cooling function.

Apparently, the gas cloud would loose its energy due to radiative cooling and would collapse to the central singularity in a finite time, the process known as the "cooling catastrophe". In order to simulate this process, the evolution of the gas is followed numerically by a one-dimensional Lagrangian code as described in Gnedin (1995) (adapted for the spherically symmetric case and combined with the exact gravity solver). Since the hydrodynamic solver is exactly Lagrangian, the gravitational consistency condition becomes trivial (Gnedin & Bertschinger 1996).

At each hydrodynamic time step t_n new values of the gas density ρ_n, velocity v_n, and temperature without cooling $\tilde{T}_n$ are computed using the values at the previous time step t_{n-1}. Then, in order to account for radiative cooling, the following equation for the temperature is solved:

$$\frac{\mathrm{d}T}{\mathrm{d}t} = -(\gamma-1)\rho_n \Lambda(T) \tag{4}$$

in a time interval $\Delta t = t_n - t_{n-1}$ with the initial condition $T(t_{n-1}) = \tilde{T}_n$.⋆ This is the way the radiative cooling is incorporated in most of existing cosmological hydrodynamic codes.

Figure 1 shows the gas density, velocity and temperature as a function of radius at two different moments: the initial moment $t = 0$, shown as a thin solid line, and the moment when the central density reached 10^{12}, i.e. increased by 12 orders of the magnitude (the bold line). At this moment the simulation with 256 zones encountered numerical problems at the center due to the round-off error in double precision calculations. The density distribution is a perfect power law with the index -2 across the 7 orders of magnitude in radius and 14 orders of magnitude in density, and the gas temperature is almost constant, changing by less than 0.2 dex (60%) across 7 orders of magnitude in radius.

This results demonstrates both the success and failure of the purely Lagrangian gas dynamics. It definitely succeeds in achieving enormous dynamical range, and simultaneously it fails to carry on the solution beyond the collapse time. Indeed this is a correct result which tells one that the simple model one adopted is not sufficient to describe the behavior

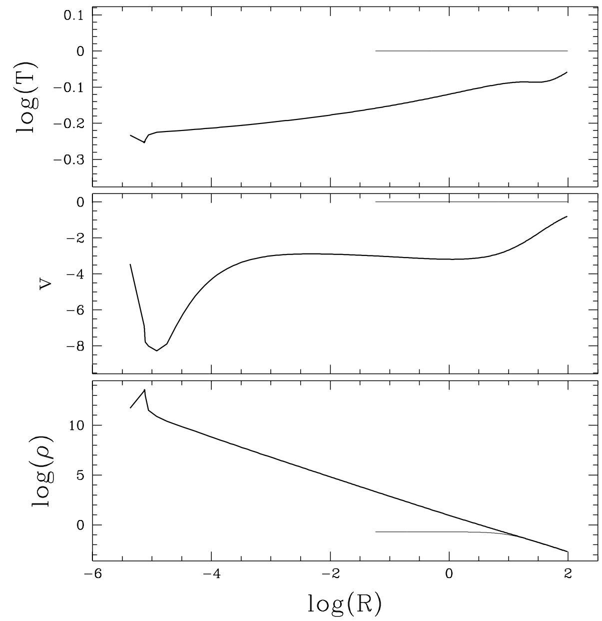

Figure 1. The gas density (*lower panel*), velocity (*middle panel*), and temperature (*upper panel*) for the exact spherically symmetric collapse as a function of radius. Two moments are shown: the initial moment (*thin line*) and the final moment (*bold line*) defined as the moment when the central density has increased by 12 orders of magnitude. The density profile is a perfect power-law with the index -2 (except near the center where numerical loss of accuracy occurs at very high densities).

⋆ With the cooling function (3) this equation can be solved analytically.

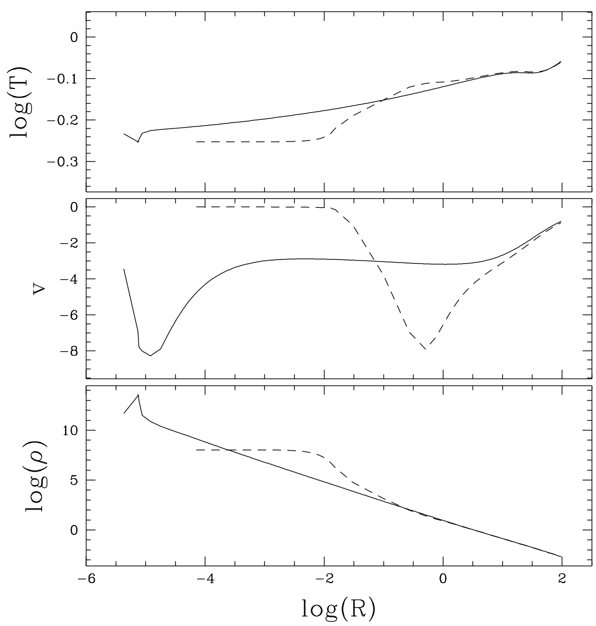

Figure 2. The gas density (*lower panel*), velocity (*middle panel*), and temperature (*upper panel*) as a function of radius for the exact case (*solid line*) and the softened gravity case (*dashed line*). The softened case is shown for the time instant $t = 1.1 t_{\rm col}$, where $t_{\rm col}$ is the collapse time of the exact solution.

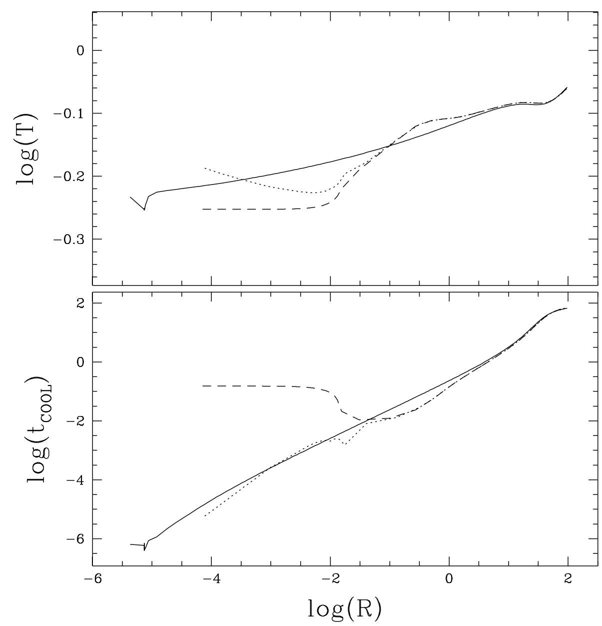

Figure 3. The gas cooling time (*lower panel*) and temperature (*upper panel*) for three cases: exact solution (*solid line*), softened gravity without cooling consistency condition (*dashed line*), and softened gravity with the cooling consistency condition (*dotted line*). The balance between heating and cooling is broken in the softened gravity solution which is manifested in the unphysically long cooling times; the balance is restored back in the consistent solution.

of a real physical system. Nevertheless, in a real three dimensional simulation, one would like to proceed further in time simulating other regions of the universe even if there has formed a real singularity in one point in space. Numerically this is achieved by softening the gravity solver, i.e. using a softened gravity law instead of the exact $1/r^2$ Newtonian law.

Let me now consider the cooling catastrophe for the softened gravity case. The standard Plummer softened gravity law is adopted with the softening length $\epsilon = 0.01$ and the same initial configuration is followed forward in time. In this case no real singularity forms, and the solution can be carried on in time indefinitely. Figure 2 shows the gas density, velocity and the temperature for the exact case as in Fig.1 with a solid line, and for the softened case with the dashed line. The softened solution is shown at a moment $t = 1.1 t_{\rm col}$, where $t_{\rm col}$ is the time of collapse for the exact solution. As one can expect, the velocity approaches zero, and the density and the temperature approach finite values at the center, with the core radius close to the softening length ϵ.

As one can expect, the softened solution provides a good approximation to the real solution on scales larger than several softening lengths. The temperature inside the softening length is only slightly lower than the exact solution and thus we may conclude that the softened solution is a good approximation to the exact solution. Here, however, the cooling consistency condition steps in.

When the gravitational consistency condition is violated, there is a simple basic law of physics - energy conservation - which is also violated, and it is then clear that the gravitational consistency condition has to assure the conservation of energy. It is not quite so with the cooling consistency condition, since there is no basic physical law that is violated by the softened solution in Fig.2. If, for example, one is satisfied with the mere existence of the gas globs described by the dashed line in Fig.2 in one's simulation, one can simply ignore any cooling inconsistency that may appear in the softened solution. However, if one does not restrict himself to the mere existence of those gas globs, but is interested whether those objects actually cool and collapse to eventually form stars (as the exact solution does), the softened solution becomes unacceptable. In Figure 3 I show the cooling time and the temperature as a function of radius for the exact solution (the solid line) and the softened solution (the dashed line; the dotted line is explained below). Even if temperatures of two solutions differ by a small amount, the cooling function is so steep, that the cooling time of the softened solution exceed the cooling time of the exact solution by many orders of magnitude. At every point in the exact solution the cooling time $t_{\rm COOL}$ is similar to the local dynamical time $t_{\rm DYN}$,

$$t_{\rm COOL} \sim t_{\rm DYN}.$$

This condition is satisfied because the collapse proceeds at the dynamical time, and if the cooling time is much shorter than the dynamical time, the temperature will quickly decrease, the cooling function will also decrease and the cooling time will increase until it is close to the dynamical time; if, inversely, the dynamical time is shorter than the cooling time, the temperature will increase adiabatically, the cooling function will increase and the cooling time will decrease until it is again approximately equal to the dynamical time. The collapse is therefore proceeding in the quasi-equilibrium

state, when adiabatic heating is approximately balanced by radiative cooling.

The softened solution, however, grossly violates this quasi-equilibrium. Since the softened gravity prevents the collapse much beyond the resolution limit of the code, there is no adiabatic heating to balance cooling, the gas temperature will continue to decrease and the cooling time will continue to increase as long as the simulation progresses, so that at a sufficient late time moment t,

$$t_{\rm COOL} \sim t \gg t_{\rm DYN}.$$

Then, if one tries to estimate whether the object is actually cooling and collapsing by comparing its dynamical time to its cooling time, one would find that the cooling time is several orders of magnitude larger than the dynamical time, and would erroneously conclude that the object does not cool and, therefore, does not collapse!

This conclusion stems from the fact that the hydrodynamic and cooling solvers are not consistent with each other in a sense that the cooling solver (4) knows nothing about whether the resolution limit of the code has been already reached or not. I can therefore immediately conclude that for a fully Lagrangian solver the cooling consistency condition is trivial as is the gravitational consistency condition. However, in this case it is assumed that the fully Lagrangian solver obeys the $1/r^2$ gravity law, whereas for the gravitational consistency condition only potentiality of the force is required (Gnedin & Bertschinger 1996).

Let me now consider the softened case and address the question whether we can repair the deficiency of the cooling solver and make it consistent with the (self-gravitating) hydrodynamic solver. Since the inconsistency appears as the "overcooling" problem when the resolution limit is reached, the obvious solution is to reduce the cooling rate at the resolution limit of the code. I therefore replace the radiative cooling term (2) with the following expression:

$$\left(\frac{{\rm d}U}{{\rm d}t}\right)_{\rm COOL} = -D\rho^2\Lambda(T), \tag{5}$$

where D is the *cooling consistency factor*, $0 < D < 1$. Obviously, D should be a function of the softening length ϵ, so that $D = 1$ when $r \gg \epsilon$ and $D = 0$ if $r \ll \epsilon$. To demonstrate the effect of the cooling consistency condition, I adopt the following form for the cooling consistency factor D:

$$D(r,\epsilon) = \left(\frac{r^2}{r^2 + a\epsilon^2}\right)^b, \tag{6}$$

where

$$a = \begin{cases} 1, & \text{if } r < 0.1\epsilon, \\ 0.01, & \text{if } r > \epsilon, \text{ and} \\ 0.01(\epsilon/r)^2, & \text{otherwise,} \end{cases} \tag{7}$$

and

$$b = \begin{cases} 0.75, & \text{if } r < 0.01\epsilon, \\ 0.85, & \text{if } r > 0.1\epsilon, \text{ and} \\ 0.75 + 0.1(2 + \log_{10}(r/\epsilon)), & \text{otherwise.} \end{cases} \tag{8}$$

The dotted line in Fig.3 shows the temperature and the cooling time for the softened case when the cooling consistency condition is incorporated. The cooling time agrees very well with the exact solution. The density and the velocity profiles for the corrected case coincide with the uncorrected softened case almost precisely, so the cooling consistency condition does not change the density profile. Apparently, it is possible to achieve even better agreement by an appropriate choice of the cooling consistency factor D. The particular choice presented above serves merely to demonstrate the importance and effect of the cooling consistency condition.

3 COOLING CONSISTENCY CONDITION IN THREE DIMENSIONS

Let me first briefly discuss the rationale behind introducing the cooling consistency factor D. The equation for the temperature of a fluid element with a constant polytropic index γ can be written as:

$$\frac{{\rm d}T}{{\rm d}t} = (\gamma - 1)\left[\frac{T}{\rho}\frac{{\rm d}\rho}{{\rm d}t} - \rho\Lambda(T,\rho)\right], \tag{9}$$

where d/dt denotes the Lagrangian time-derivative, and the cooling function Λ is not required to be a function of T only. For a cooling catastrophe, the dynamical time is close to the cooling time, which implies that the two terms in square brackets almost cancel each other and the collapse is almost isothermal:

$$\frac{T}{\rho}\frac{{\rm d}\rho}{{\rm d}t} \approx \rho\Lambda(T,\rho). \tag{10}$$

When equation (9) is implemented numerically, the time derivative becomes numerical time derivative for a finite resolution numerical scheme, which I emphasize by adding a subscript N to it:

$$\frac{{\rm d}_N T}{{\rm d}t} = (\gamma - 1)\left[\frac{T}{\rho}\frac{{\rm d}_N\rho}{{\rm d}t} - \rho\Lambda(T,\rho)\right]. \tag{11}$$

The main difference between equations (9) and (11) is that at the resolution limit of the code the density stops changing,

$$\frac{{\rm d}_N\rho}{{\rm d}t}(\text{res. limit}) \to 0, \tag{12}$$

and equation (11) becomes:

$$\frac{{\rm d}_N T}{{\rm d}t} = -(\gamma - 1)\rho\Lambda(T,\rho), \tag{13}$$

implying that the cooling time will increase in proportion to the physical time t and not the dynamical time $t_{\rm DYN}$. The equation (11) can be "repaired" by introducing the cooling consistency factor D,

$$\frac{{\rm d}_N T}{{\rm d}t} = (\gamma - 1)\left[\frac{T}{\rho}\frac{{\rm d}_N\rho}{{\rm d}t} - D\rho\Lambda(T,\rho)\right]. \tag{14}$$

Now, by requiring that

$$D = \frac{{\rm d}_N\rho/{\rm d}t}{{\rm d}\rho/{\rm d}t}, \tag{15}$$

I restore the quasi-equilibrium character of the collapse (eq. [10]). The only disadvantage of this approach is that the exact cooling consistency condition (15) can never be satisfied since the exact time derivate of the density, $d\rho/dt$ is unknown, and more that that, it is what one tries to approximate with the numerical derivative ${\rm d}_N\rho/{\rm d}t$. The cooling consistency condition can therefore be satisfied only *approximately*.

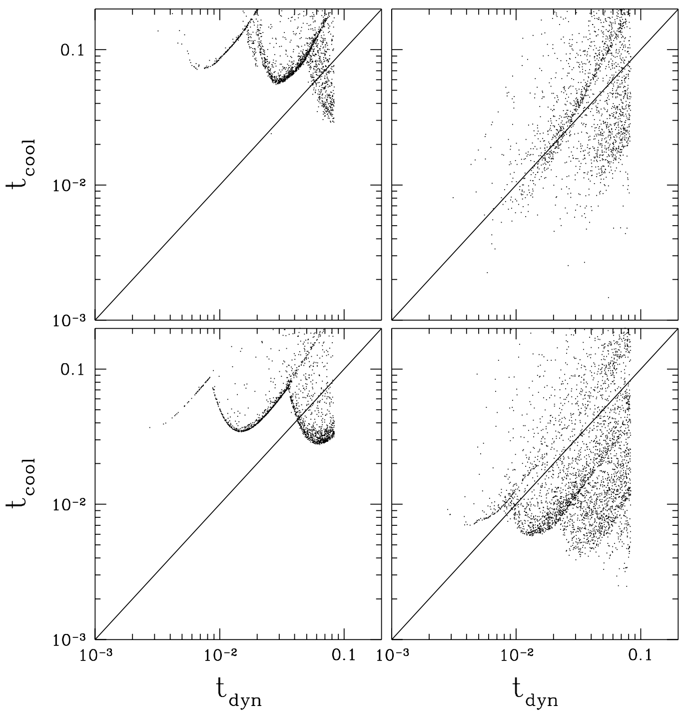

Figure 4. The gas cooling time vs dynamical time (in arbitrary units) for realistic three-dimensional cosmological simulations of a standard CDM model: a 160 dynamical range simulation with 5% baryons and without the cooling consistency condition (*upper left panel*); a 160 dynamical range simulation with 50% baryons and without the cooling consistency condition (*lower left panel*); a 160 dynamical range simulation with 5% baryons and with the cooling consistency condition (*upper right panel*); a 640 dynamical range simulation with 5% baryons and without the cooling consistency condition (*lower right panel*); Again, the balance between heating and cooling is broken without the consistency condition independently of the baryonic fraction; the balance is restored by the cooling consistency condition or by increasing the resolution of a simulation. In the latter case however, cooling and heating rates get out of balance again at the (finer) resolution limit of a higher resolution simulation.

Reducing cooling is not the only way to achieve cooling consistency. It is possible, for example, to introduce additional heating terms in equation (11) to account for the loss of adiabatic heating near the resolution limit of the code. Therefore, there exist many different ways to satisfy the cooling consistency condition, and the cooling consistency factor D is only one of them.

Let me now turn to a three-dimensional case. I use the SLH-P^{3}M code (Gnedin & Bertschinger 1996) to demonstrate the effect of the cooling consistency condition on the thermal evolution of the cosmic gas. I adopt the standard CDM model as a testbed for my demonstration. The cosmological parameters are fixed so that $\Omega_0 = 1.0$, $h = 0.5$, and I fix the initial power spectrum by using the BBKS transfer function (Bardeen et al. 1986) and adopting $\sigma_8 = 1.0$.

First, two 32^3 runs with the softening parameter of 1/5 (the dynamical range of 160) and *without* any consistency condition were performed, the first one using $\Omega_b = 0.05$, and, therefore, having 5% baryons, and the second one using $\Omega_b = 0.5$ (50% baryons) with otherwise identical initial conditions. Figure 4 shows cooling and dynamical times for all fluid elements with overdensities in excess of 100 and temperatures between $10^{3.5}\,$K and $10^{4.5}\,$K at $z = 6$ in a $2h^{-1}\,$Mpc box (at this moment most of the gas has not yet been shocked to temperatures significantly exceeding $10^4\,$K) for those two runs in the upper left and lower left panels respectively. It is apparent that the cooling time in the highest density regions is very long, exceeding the dynamical time by up to two orders of magnitude, and the balance between the cooling and heating rates is severely broken - the manifestation of the cooling inconsistency. Were those fluid elements tested on the condition of collapse, one would find that they cannot cool, and, therefore, cannot collapse, which is apparently nonphysical conclusion. As one can expect, the cooling inconsistency does not depend on the baryonic fraction ((and, hence, on the dark matter fraction) since equation (11) contains no reference to the dark matter fraction.

I can now introduce the *SLH cooling consistency condition* by defining the cooling consistency factor D in the following way:

$$D = 1 - \min(\sigma_1, \sigma_2, \sigma_3), \tag{16}$$

where σ_j are eigenvalues of the deformation tensor σ^{ij} which defines the resolution limit of the SLH code (for exact definitions see Gnedin & Bertschinger (1996)). The cooling consistency condition in this form gradually reduces the total cooling rate as the gas element followed by the SLH code approaches the code resolution limit.

Since the SLH code is not exactly Lagrangian, and, in general, the flow is not spherically symmetric, an expression for D found in the spherically symmetric case (eq. [6]) cannot be applied in a three-dimensional case exactly. However, in the spherically symmetric case, the expression (16) reduces to equation (6) with $a = b = 1$. While corrections (7) and (8) improve the accuracy of the approximate solution, they cannot be easily generalized for a three-dimensional case. The three-dimensional consistency condition (16) is, therefore, less accurate that the three-dimensional approximation given by (6); further improving upon the accuracy of the SLH consistency condition is beyond the frame of this paper.

I can now test the SLH consistency condition (16). The distribution of cooling and dynamical times for a 32^3 simulation with the 1/5 softening and $\Omega_b = 0.05$ *with* the cooling consistency condition as given by equation (16) is shown in Fig.4 at the upper right panel. One can note now that in the run with the consistency condition, the cooling times are in a reasonable (yet not perfect, since the cooling consistency condition (16) is only an approximate one) agreement with the dynamical times for all fluid elements with high density. For those fluid elements the cooling time is of the order of their dynamical time and they are *apparently* cooling and collapsing. One must remember, of course, that in the simulation most of those fluid elements have already reached their resolution limit and their density does not significantly increase with time, but the relationship between their cooling and dynamical times mimics that of truly cooling and collapsing gas clouds. A simulation with the cooling consistency condition and $\Omega_b = 0.5$ (50% baryons) produces nearly identical distribution of cooling and dynamical times and is not shown here due to the space limitations.

Finally, the lower right panel of Fig.4 shows the distribution of cooling and dynamical times for a 64^3 simulation with the softening parameter of 1/10 (the dynamical range of 640) with the same $2h^{-1}$ Mpc box and $\Omega_b = 0.05$ *without* the cooling consistency condition (for clarity only every eights fluid element is shown). Since this simulation has four times higher spatial resolution than the simulation shown in the upper left panel of Fig.4, the balance between cooling and heating extends to about a factor of 64 higher densities, or, equivalently, to about a factor of 8 smaller dynamical times. One can easily see that the simulation with the consistency condition (the upper right panel) mimics very closely the high resolution simulation. At the highest densities the balance is again broken, as manifested by the cooling time becoming larger than the dynamical time as the dynamical time decreases. The effect is not as dramatic as in the upper left panel, and the gas densities in the high resolution simulation are only slightly larger than in the low resolution simulations because the final epoch ($z = 6$) was chosen such that in the high resolution simulation only a small number of all fluid elements have actually reached their resolution limit; this makes the high resolution simulation closely mimicking an imaginary exact (infinite resolution) case.

4 CONCLUSIONS

I have shown on simple spherically symmetric and realistic three-dimensional examples how the inconsistency between the finite resolution self-gravitating hydrodynamic solver and the radiative cooling term may arise. For a efficiently cooling and collapsing gas clouds the cooling time approximately balances the dynamical time so that the collapse is occurring in the quasi-equilibrium between adiabatic heating and radiative cooling.

In simulations, the finite resolution leads to decrease in adiabatic heating when a fluid elements approaches the resolution limit of a numerical code, and the balance between heating and coolings breaks down. This leads to an "overcooling" problem, when the cooling time in the fluid element becomes significantly longer than the dynamical time, and the fluid element appears cooling very inefficiently.

The contradiction is eliminated when the cooling consistency condition is introduced, which reduces the cooling rate in proportion to the reduction in the heating rate. I present the exact cooling consistency condition, which however cannot be realized in practice since it depends on the unknown true solution which a simulations tries to approximate. However, approximate cooling consistency conditions can be introduced both in a spherically symmetric and realistic three-dimensional cases which are "good enough" in a sense that the resultant approximate numerical solution mimics the exact solution in balancing the heating and cooling rates.

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